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EXPERIMENTAL DETERMINATION OF THE DIFFUSION LENGTH FOR C NEUTRONS IN BeO

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ABSTRACT

The diffusion length for C neutrons in BeO is determined experimentally to be 29.4 cm. The experiment consists of inserting a 6" layer of BeO in a sigma pile. The saturated activities of indium foils placed in a slot above the BeO layer are compared with the activities of foils placed in the same slot, the BeO having been replaced by graphite. After determining the diffusion length in graphite the diffusion length in BeO is calculated from this ratio.

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EXPERIMENTAL DETERMINATION OF THE DIFFUSION LENGTH FOR C NEUTRONS IN BeO

Because the diffusion length[1] of thermal neutrons in BeO (the tamper material of the water boiler[2]) enters into the theoretical calculations for the critical mass of the boiler, it was considered of interest to determine this quantity experimentally. The measurement was made by comparing, in a sigma pile, the saturated activities of indium foils with and without a layer of BeO. Since the value obtained for BeO by this method depends directly on the diffusion length in the particular graphite used, it was necessary first to determine the diffusion length in the AGOT pile graphite. This also served as a useful check on the experiment since it is known that the diffusion length in AGOT graphite should be approximately 50 cm.

Graphite

The graphite measurement was made in essentially the usual way (report Cl-573, Fig. 2), by inserting a layer of cadmium across the pile at a level 34.3 cm above a 200 mc RaBe source placed at the center of a plane 15-3/8" above the bottom of the pile. Other pile dimensions are listed at the end of this report. The saturated activities of indium foils placed vertically above the source in slots z_I, z_{II}, z_{III}, respectively 55.8 cm, 77.17 cm, and 120.48 cm above the source were determined with and without the Cd layer. In each slot the difference of the two activities

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1) Diffusion length is defined as the distance at which the neutron intensity falls off by a factor 1/e in an infinite medium with a plane source or qualitatively described as the shortest distance from birth to capture of a therman neutron.

2) LA-134

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measures the density at that point without Cd, of those neutrons which would be captured in the cadmium layer. In this way the plane of the layer acts essentially as a slow-neutron "source"; actually, the diffusion length measured is that of C neutrons.

The calculations were complicated considerably by the placing of the RaBe source in the center of the source plane (i.e., at coordinates $x = 0$, $y = 0$) instead of at a point half-way between the center and one edge of the pile (i.e., $x = a/4$, $y = 0$). Since the harmonic corrections are very large in this case, the delta function approximation to the thermal source (as used in CL-573) is not sufficiently accurate. Instead it was found necessary to take for the actual thermal density at the cadmium layer the exact solution of the neutron diffusion equation given by Fermi in reports A-21, and A-6. Using this solution at the Cd layer as a source function for the neutrons measured, and assuming that they diffuse exponentially to the top of the pile at which point the density is zero, the following equation is obtained for the neutron density at any point z along the pile axis above the Cd layer:

$$n(z) = \sum_{\ell,m=1}^{\infty} \frac{1}{2c_{\ell m}} \sum_{i=1,2,3} f_i \, e^{-3r^2/4\lambda^2_N} \left[\left\{A - \Theta\left(\frac{z_o}{r_i} + \frac{r_i}{2c_{\ell m}}\right)\right\} e^{2z_o/c_{\ell m}} \right. \tag{1}$$

$$\left. + \left\{B + \Theta\left(\frac{z_o}{r_i} - \frac{r_i}{2c_{\ell m}}\right)\right\}\right] e^{-z/c_{\ell m}} = X_{\ell m} \, c_{\ell m} \, e^{-z/c_{\ell m}}$$

where z_o = the distance from source to Cd layer

$c_{\ell m}$ = relaxation distance, i.e.: distance in which the intensity drops off by a factor $1/e$

$A = 1.000$

$B = .9316$

$$\Theta(x) = (2/\sqrt{\pi}) \int_0^x e^{-\lambda^2} \, dx$$

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N = number of impacts before capture

r = the average distance from the RaBe source to the point at which the neutron becomes thermal

The values of f_i and r_i were taken from report A-6 (Eq. 7) as:

i	f_i	r_i(thermal)
1	.14	27.1
2	.658	39.8
3	.202	58.9

Eq. (1) neglects a very small additional term due to the finite height of the pile.

If the density fell off with distance from the source as a single exponential, then the density at any two points would be given by

$$n_1 = Ae^{-z_1/c_{11}}$$

$$n_2 = Ae^{-z_2/c_{11}}$$

solving for c_{11} one would obtain

$$\frac{1}{c_{11}} = \frac{1}{z_2 - z_1} \log \frac{n_1}{n_2}$$

In order to correct this equation for the actual case in which there is more than one harmonic, each density must be multiplied by the ratio of the first harmonic to the sum of all the harmonics, i.e.,

$$\frac{c_{11}\, X_{11}\, e^{-z/c_{11}}}{\sum\limits_{\ell m=1}^{\infty} c_{\ell m}\, X_{\ell m}\, e^{-z/c_{\ell m}}} = E(z)$$

The equation for c_{11} combining any two slots z_I and z_{II} then becomes

$$\frac{1}{c_{11}} = \frac{1}{z_{II} - z_I} \log \left[\frac{\left(A_s(\text{no Cd}) - A_s(\text{Cd})\right)_{z_1}}{\left(A_s(\text{no Cd}) - A_s(\text{Cd})\right)_{z_2}} \; \frac{E(z_1)}{E(z_2)} \right]$$

were A_s = saturated activity of the indium foils. Knowing c_{11}, the diffusion length L can be calculated by the relation

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$$\frac{1}{L^2} = \frac{1}{c_{11}^{\,2}} - \frac{2\pi^2}{a^2}$$

where a = width of the pile.

Since the large harmonic correction applied to slot I seemed less reliable than those applied to slots two and three, only data combining slots II and III were used to determine the final value of c_{11}. The value so obtained was 28.44 cm giving a corresponding diffusion length of 48.4 cm for the particular graphite used.

BeO

In order to determine the diffusion length in BeO, a 6" layer of BeO was placed between slots II and III. Since BeO has better neutron reflecting properties than graphite, the change in neutron distribution as a function of distance from the source looks qualitatively somewhat like Fig. 1.

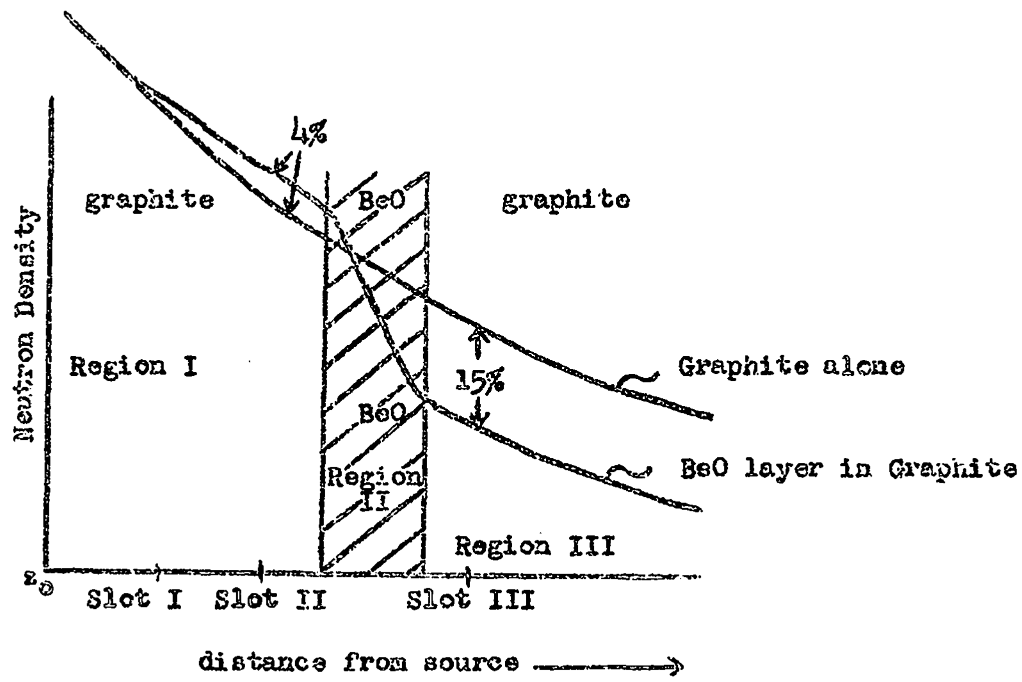

Fig. 1

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Christy derived equations for a similar sandwich arrangement. These equations give at any point the ratio of the neutron density with the sandwich layer to that without it. Knowing this ratio, the diffusion length can be calculated since the ratio is a function of the relaxation distance in the sandwich material.

As by far the largest effect is observed above the BeO layer, the calculated ratio for this region only was used; however, with more accurate data it might be possible to distinguish between absorption and scattering cross section by comparing the ratios below to those above the BeO layer. The general equations for the three regions of the sandwich are:

$$\text{R-I} \qquad n = \sum A_{\ell m}\left(e^{-z/c_{\ell m}} + B_{\ell m}\, e^{z/c_{\ell m}}\right)$$

$$\text{R-II} \qquad n = \sum C_{\ell m}\left(e^{-z/b_{\ell m}} + D_{\ell m}\, e^{z/b_{\ell m}}\right)$$

$$\text{R-III} \qquad n = \sum E_{\ell m}\left(e^{-z/c_{\ell m}} + F_{\ell m}\, e^{z/c_{\ell m}}\right)$$

where $c_{\ell m}$ is the relaxation distance in carbon and $b_{\ell m}$ is the relaxation distance in BeO. Using the boundary conditions that both flow and density are continuous at the boundaries between graphite and BeO, that the density is zero at z_3 and equal to some source function $N(o)_{\ell m}$ at z_o, all the constants can be evaluated in terms of

$$N(o)_{\ell m},\ c_{\ell m},\ b_{\ell m},\ z_1,\ z_2,\ z_3,\ \lambda_{BeO},\ \lambda_C$$

On taking the ratio of BeO to carbon the source function drops out leaving $b_{\ell m}$ as the only unknown.

$$\frac{\text{Intensity}_{III}\text{BeO}}{\text{Intensity}_{III}\text{C}} = \sum \frac{\sinh(z_3/c_{\ell m})\, e^{-z_1/c_{\ell m}}\, e^{-(z_2-z_1)/b_{\ell m}}\,(1+\beta)\,(1-\alpha)}{\sinh\left[(z_3-z_2)/c_{\ell m}\right]\left(1-\alpha\, e^{-2(z_2-z_1)/b_{\ell m}}\right)\left(1+\beta\, e^{-2z_1/c_{\ell m}}\right)}$$

where

$$\alpha = \frac{\left[1-(\lambda_{BeO}/\lambda_C)(c_{\ell m}/b_{\ell m})\tanh\left\{(z_3-z_2)/c_{\ell m}\right\}\right]}{\left[1+(\lambda_{BeO}/\lambda_C)(c_{\ell m}/b_{\ell m})\tanh\left\{(z_3-z_2)/c_{\ell m}\right\}\right]} = .1488$$

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and

$$\beta = \frac{(\lambda_C/\lambda_{BeO})(b_{\ell m}/c_{\ell m})\left[(1-\alpha\, e^{-2(z_2-z_1)/b_{\ell m}})/(1+\alpha\, e^{-2(z_2-z_1)/b_{\ell m}})\right] - 1}{(\lambda_C/\lambda_{BeO})(b_{\ell m}/c_{\ell m})\left[(1-\alpha\, e^{-2(z_2-z_1)/b_{\ell m}})/(1+\alpha\, e^{-2(z_2-z_1)/b_{\ell m}})\right] + 1} = 0.1008$$

With this equation and the experimental ratio $(\text{Intensity}_{III}\text{BeO})/(\text{Intensity}_{III}\text{C})$, by use of only the first harmonic, b_{11} can be obtained by the method of successive approximation. Since slot III was 1-3/4" higher with BeO than without it, the graphite data were multiplied by an exponential factor making the two intensities correspond to the same position. The final result obtained for b_{11} was 22.5 cm giving a diffusion length of 29.4 cm. The calculated value from the equation $L = 1/(N\sqrt{3\sigma_a\sigma_t})$ using the most recent cross sections given by Uchiymaba (i.e., $\sigma_t = 8.2$, $\sigma_a = .0115$) is 29.0 cm. The excellent agreement must be considered fortuitous.

The indium foils used for this experiment were of the standard Chicago type 5 mils thick, 4 x 6.5 cm. Data obtained with these foils were reproducible to within 2%. To determine A_s three Pb-shielded Geiger counters with scales of 64 were used. The set factors between the units were determined by counting the same foil in each counter and comparing the saturated activities obtained. In the experiment all foils were counted to 20,000 counts, at least three foils being taken in each position.

An 8% correction was applied to all cadmium activities to account for the absorption in Cd of neutrons in the indium resonance energy region.

If the graphite value is assumed correct the BeO should be correct to within approximately 3%; however, an estimate of the errors in the graphite value is hard to make as it depends mainly on the calculated harmonic correction. The maximum spread in the values of L obtained from the three possible combinations of data using all three slots is ~7%; however, increasing the harmonic correction to slot 1 from the calculated value of 9% up to 11% decreases the maximum spread to less than ½%. For

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large harmonic corrections applied to points near the source such as slot I it is conceivable that the calculations may not be sufficiently accurate. For example, a considerable error might be introduced by the fact that the values of r_λ used were not those measured for the particular source and graphite used. As already stated it was for this reason that slots II and III only were used to determine C_{11}.

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PILE DIMENSIONS

BeO

z_0 = 34.3 cm

z_I = 55.58

z_{II} = 77.17

z_{III} = 125.00

z_1 = 88.38

z_2 = 103.68

z_3 = 212.53

Density BeO = 2.69

Graphite

z_0 = 34.3 cm

z_I = 55.58

z_{II} = 77.17

z_{III} = 120.48

z_3 = 208.01

a = 156.0

with $2\lambda/\sqrt{3}$ correction

Density of graphite $\approx$ 1.61

DATA

	$\bar{A}_s$ Graphite	$\bar{A}_s$ BeO
Slot I		
no Cd	6873	
Cd	1768	
Cd diff	5105	
Slot II		
no Cd	3186	
Cd	942	
Cd diff	2244	
Slot III		
no Cd	682.1	493.4
Cd	205.6	146.4
Cd diff	476.5	347

Valves for L_c using different slot combinations

I + III = 48.44

I + III = 47.7

I + II = 45.2

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